



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

**MATHEMATICS P2/WISKUNDE V2
NOVEMBER 2019
MARKING GUIDELINES/NASIENRIGLYNE**

MARKS/PUNTE: 150

**These marking guidelines consist of 26pages.
Hierdie nasienriglyne bestaan uit 26 bladsye.**

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • MEETKUNDE	
S	A mark for a correct statement (A statement mark is independent of a reason)
	<i>'n Punt vir 'n korrekte bewering</i> (<i>'n Punt vir 'n bewering is onafhanklik van die rede</i>)
R	A mark for the correct reason (A reason mark may only be awarded if the statement is correct)
	<i>'n Punt vir 'n korrekte rede</i> (<i>'n Punt word slegs vir die rede toegeken as die bewering korrek is</i>)
S/R	Award a mark if statement AND reason are both correct
	<i>Ken 'n punt toe as die bewering EN rede beide korrek is</i>

QUESTION/VRAAG 1

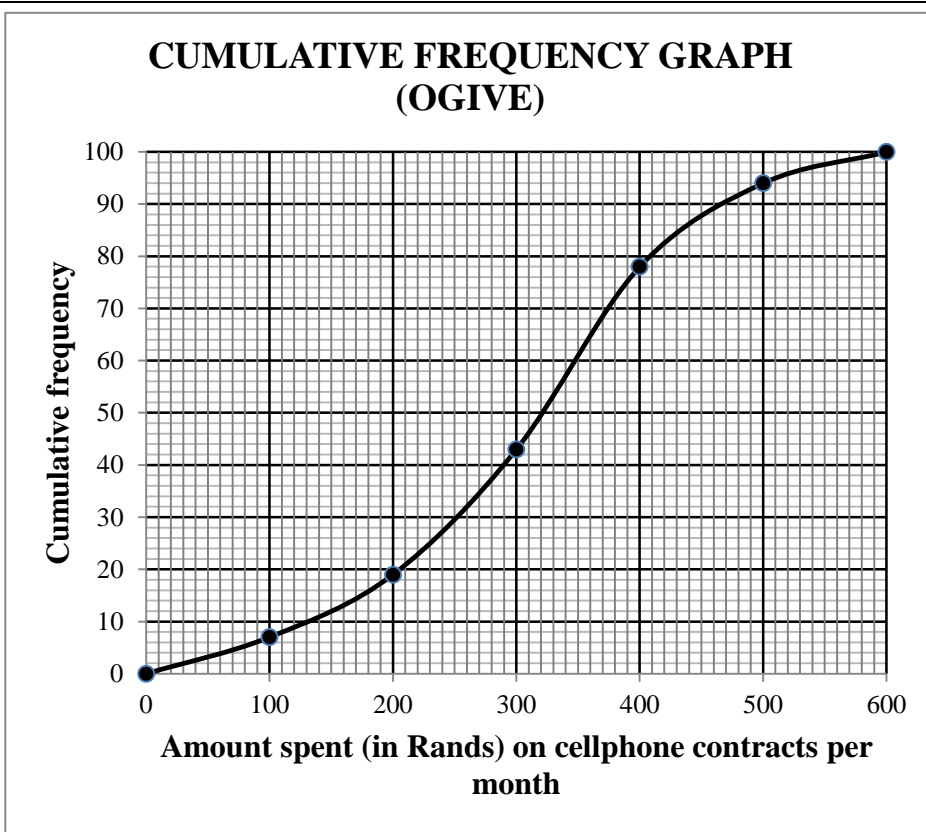
Monthly income (in rands) Maandelikse inkomste (in rand)	9 000	13 500	15 000	16 500	17 000	20 000
Monthly repayment (in rands) Maandelikse paaieiment (in rand)	2 000	3 000	3 500	5 200	5 500	6 000

1.1	$a = -1946,875... = -1946,88$ $b = 0,41$ $\hat{y} = -1946,88 + 0,41x$	Answer only: Full marks	✓ $a = -1946,88$ ✓ $b = 0,41$ ✓ equation (3)
1.2	Monthly repayment \approx R3 727,16 (calculator) <i>Maandelikse paaieiment \approx R3 727,16</i> OR $\hat{y} = -1946,88 + 0,41(14000)$ \approx R3 793,12		✓✓ answer (2) ✓ substitution ✓ answer (2)
1.3	$r = 0,946 \dots \approx 0,95$		✓ answer (1)
1.4	Not to spend R9 000 per month because the point (18 000 ; 9 000) lies very far from the least squares regression line. OR D <i>Spandeer nie R9 000 per maand nie, want die punt (18 000 ; 9 000) lê baie ver van die kleinste-kwadrade regressielyn. OF D</i>		✓✓ answer (2)
			[8]

QUESTION/VRAAG 2

2.1	Number people paid R200 or less = 19 <i>Aantal mense wat R200 of minder betaal het = 19</i>	✓ answer (1)
2.2	$7 + 12 + a + 35 + b + 6 = 100$ $a = 40 - b$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times (40 - b)) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ $350 + 1800 + 10000 - 250b + 12250 + 450b + 3300 = 30900$ $200b = 3200$ $b = 16$ $a = 24$ <p>OR/OF</p> $7 + 12 + a + 35 + b + 6 = 100$ $b = 40 - a$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times b) + (550 \times 6)}{100}$ $309 = \frac{(50 \times 7) + (150 \times 12) + (250 \times a) + (350 \times 35) + (450 \times (40 - a)) + (550 \times 6)}{100}$ $350 + 1800 + 250a + 12250 + 1800 - 450a = 30900$ $200a = 4800$ $a = 24$ $b = 16$	✓ $\sum x = 100$ ✓ $a = 40 - b$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n} = 309$ ✓ $200b = 3200$ (5) ✓ $\sum x = 100$ ✓ $b = 40 - a$ ✓ $\sum fX$ ✓ $\sum \frac{fX}{n} = 309$ ✓ $200a = 4800$ (5)
2.3	Modal class/modale klas: $300 < x \leq 400$	✓ answer (1)

2.4



- ✓ grounded at (0 ; 0)
- ✓ (600 ; 100)
- ✓ cumulative frequencies for y-coordinates
- ✓ smooth shape

(4)

2.5

Number of people/Aantal mense = $100 - 82$ [accept 80 – 84 people]
 18 people paid more than R420 per month/. [accept 16 – 20 people]
 18 mense betaal meer as R420 per maand

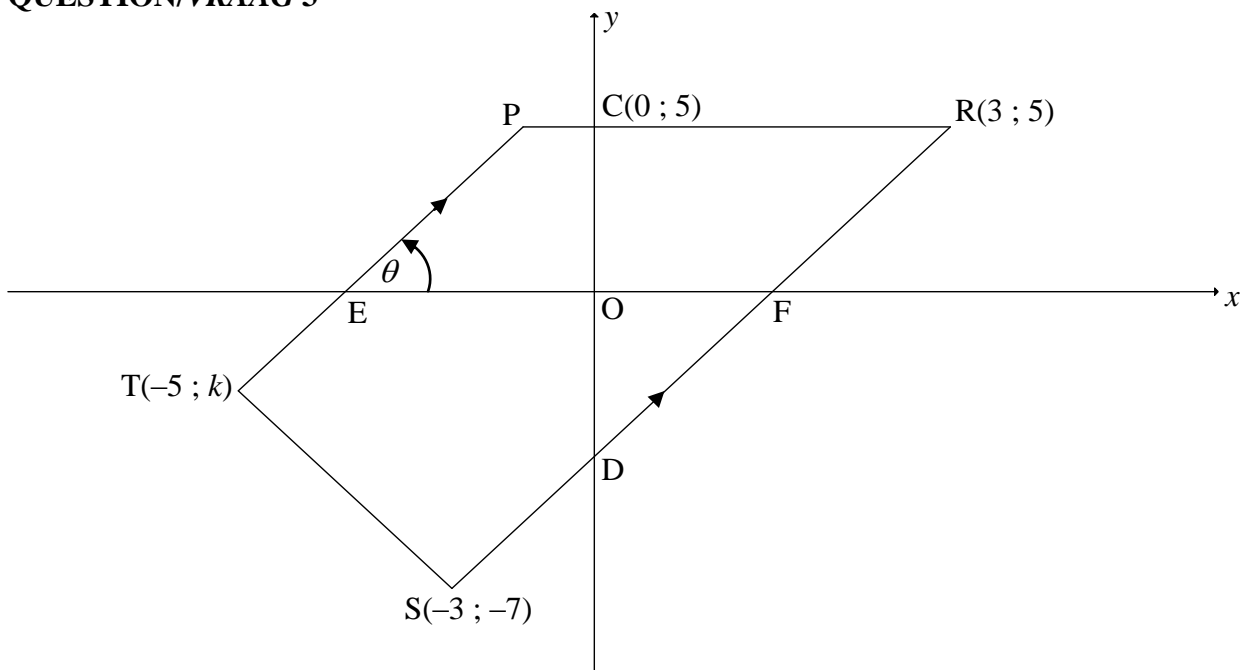
Answer only: Full marks

- ✓ 82
- ✓ answer

(2)

[13]

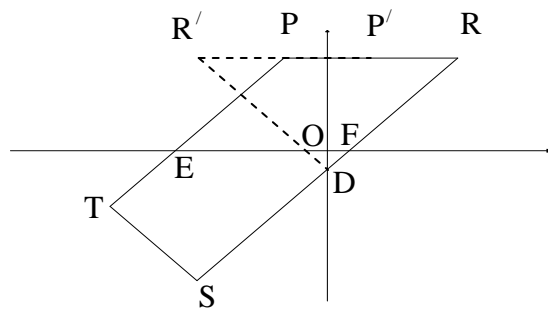
QUESTION/VRAAG 3



3.1	Equation of PR: $y = 5$	✓ answer (1)
3.2.1	$m_{RS} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{RS} = \frac{5 - (-7)}{3 - (-3)} = \frac{12}{6}$ $= 2$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px 0;">Answer only: Full marks</div>	✓ substitution of R & S into gradient formula ✓ answer (2)
3.2.2	$m_{RS} = m_{PT}$ [PT RS] $\tan \theta = 2$ $\theta = 63,43^\circ$	✓ $m_{RS} = m_{PT}$ ✓ $\tan \theta = 2$ ✓ $\theta = 63,43^\circ$ (3)
3.2.3	Equation of RS: $y - 5 = 2(x - 3)$ or $y - (-7) = 2(x - (-3))$ or $5 = 2(3) + c$ $y - 5 = 2x - 6$ $y + 7 = 2x + 6$ $c = -1$ $y = 2x - 1$ $y = 2x - 1$ $y = 2x - 1$ $\therefore D(0; -1)$ OR/OF $m_{RS} = m_{RD} = m_{DS}$ $2 = \frac{5 - y}{3 - 0} = \frac{y + 7}{0 - (-3)}$ $\therefore y = -1$ $\therefore D(0; -1)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px 0;">Answer only: Full marks</div>	✓ substitution ✓ equation of RS ✓ coordinates of D (3) ✓ equating gradients ✓ value of y ✓ coordinates of D (3)

<p>3.3</p>	$ST = 2\sqrt{5} = \sqrt{[-5 - (-3)]^2 + (k - (-7))^2}$ $20 = 4 + (k + 7)^2$ $(k + 7)^2 = 16$ $k + 7 = \pm 4$ $k = -11 \text{ or } k = -3$ $\therefore k = -3$ <p>OR</p> $ST = 2\sqrt{5} = \sqrt{[-5 - (-3)]^2 + (k - (-7))^2}$ $20 = 4 + k^2 + 14k + 49$ $k^2 + 14k + 33 = 0$ $(k + 11)(k + 3) = 0$ $k = -11 \text{ or } k = -3$ $\therefore k = -3$	<ul style="list-style-type: none"> ✓ substitute S and T into distance formula ✓ isolate square ✓ square root both sides ✓ answer <p style="text-align: right;">(4)</p> <ul style="list-style-type: none"> ✓ substitute S and T into distance formula ✓ standard form ✓ factors ✓ answer <p style="text-align: right;">(4)</p>
<p>3.4</p>	<p>Method: translation T→S:</p> $(x; y) \rightarrow (x + 2; y - 4)$ <p>∴ by symmetry: D→N: D(0 ; -1) → N(0 + 2 ; -1 - 4) ∴ N(2 ; -5)</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin-left: auto; margin-right: auto;">Answer only: Full marks</div> <p>OR</p> <p>Midpoint of TN = Midpoint of SD</p> $\frac{x + (-5)}{2} = \frac{-3 + 0}{2} \text{ and } \frac{y + (-3)}{2} = \frac{-7 + (-1)}{2}$ $x = 2 \text{ and } y = -5$ <p>∴ N(2 ; -5)</p> <div style="border: 1px solid black; padding: 2px; width: fit-content; margin-left: auto; margin-right: auto;">Answer only: Full marks</div>	<ul style="list-style-type: none"> ✓ method ✓ x-coordinate ✓ y-coordinate <p style="text-align: right;">(3)</p> <ul style="list-style-type: none"> ✓ method: midpoint of diagonals ✓ x-coordinate ✓ y-coordinate <p style="text-align: right;">(3)</p>

3.5



β is the inclination of RS $\therefore \beta = 63,434\dots^\circ$

$\hat{O}FD = 63,434\dots^\circ$ [vert opp \angle s]

$\hat{O}DF = 90^\circ - 63,434\dots^\circ = 26,565\dots^\circ$

$\hat{R}DR' = 2(26,565\dots^\circ) = 53,13^\circ$

OR

PEFR is a ||m [both pairs of opp sides ||]

$\therefore \hat{R} = \theta = 63,434\dots^\circ$ [opp \angle s of ||m]

$\hat{R}R'D = 63,434\dots^\circ$ [\angle s opp = sides: $RD = R'D$]

$\hat{R}DR' = 180^\circ - (63,43^\circ + 63,43^\circ)$ [sum of \angle s in Δ]

$\hat{R}DR' = 53,13^\circ$

OR

$\tan \hat{O}DF = \frac{3}{6}$

$\hat{O}DF = 26,565\dots^\circ$

$\hat{R}DR' = 2(26,565\dots^\circ) = 53,13^\circ$

OR

$R'(-3; 5)$ [reflection of $R(3; 5)$ about the y-axis]

$RD = \sqrt{(3-0)^2 + (5-(-1))^2}$

$RD = \sqrt{45} = R'D$ or $3\sqrt{5}$ or 6,71

$(RR')^2 = (\sqrt{45})^2 + (\sqrt{45})^2 - 2(\sqrt{45})(\sqrt{45})(\cos \hat{R}DR')$

$6^2 = 45 + 45 - 2(45)(\cos \hat{R}DR')$

$\cos \hat{R}DR' = \frac{45 + 45 - 36}{2(45)}$

$\cos \hat{R}DR' = \frac{3}{5}$

$\therefore \hat{R}DR' = 53,13^\circ$

✓ $\beta = 63,43^\circ$

✓ $\hat{O}DF = 26,57^\circ$

✓ answer

(3)

✓ $\hat{R} = 63,43^\circ$

✓ $\hat{R}R'D = 63,43^\circ$

✓ answer

(3)

✓ trig ratio

✓ $\hat{O}DF = 26,565\dots^\circ$

✓ answer

(3)

✓ $R'(-3; 5)$ **OR**

$RD = \sqrt{45} = R'D$

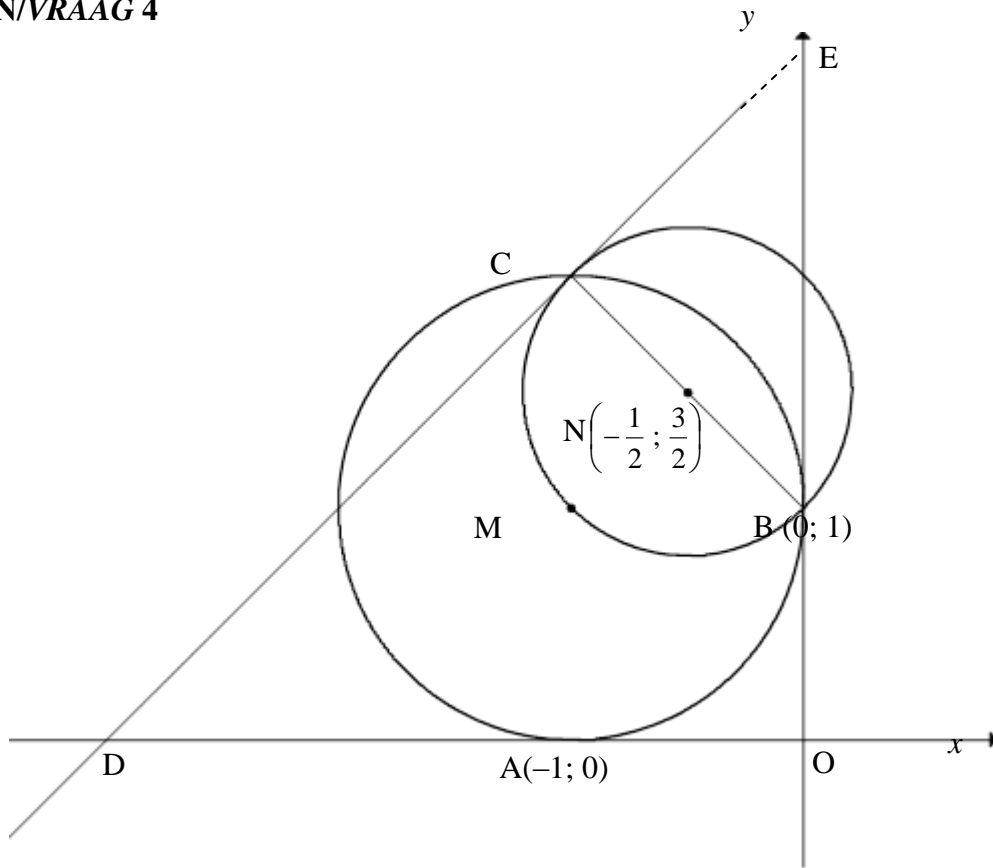
✓ substitution into cosine rule

✓ answer

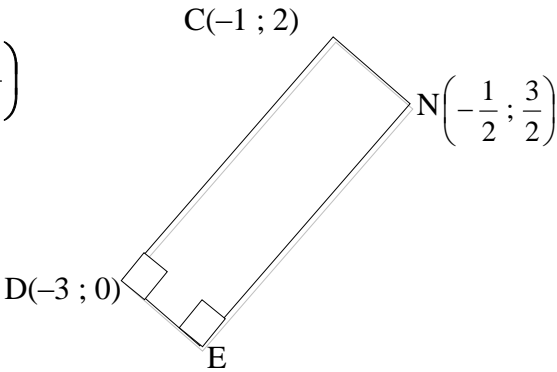
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[19]

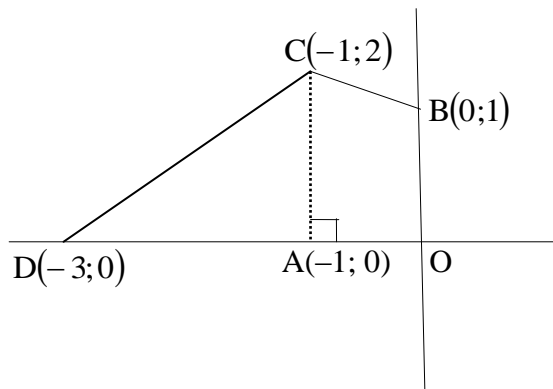
QUESTION/VRAAG 4



4.1	$M(-1;1)$ $(x+1)^2 + (y-1)^2 = 1$	<div style="border: 1px solid black; padding: 2px;">Answer only: Full marks</div>	✓ $M(-1;1)$ ✓ LHS ✓ RHS (3)
4.2	Midpoint of CB, N: $(-0,5 ; 1,5)$ $\therefore \frac{x_C + 0}{2} = -\frac{1}{2}$ and $\frac{y_C + 1}{2} = \frac{3}{2}$ $\therefore C(-1 ; 2)$ OR B \rightarrow N: $(x ; y) \rightarrow (x - 0,5 ; y + 0,5)$ N \rightarrow C: $(x ; y) \rightarrow (x - 0,5 ; y + 0,5)$ $\therefore C(-0,5 - 0,5 ; 1,5 + 0,5)$ $\therefore C(-1 ; 2)$	<div style="border: 1px solid black; padding: 2px;">Answer only: Full marks</div> <div style="border: 1px solid black; padding: 2px; margin-top: 20px;">Answer only: Full marks</div>	✓ x value ✓ y value (2) ✓ x value ✓ y value (2)

<p>4.3</p>	$m_{\text{radius}} = \frac{2-1}{-1-0} \text{ OR } \frac{2-(-\frac{1}{2})}{-1-\frac{3}{2}} \text{ OR } \frac{0-(-\frac{1}{2})}{1-\frac{3}{2}}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y = mx + c$ $y = x + c$ $2 = 1(-1) + c$ $c = 3$ $\therefore y = x + 3$ $y - x = 3$ <p>OR</p> $m_{\text{radius}} = \frac{2-1}{-1-0}$ $= -1$ $\therefore m_{\text{tangent}} = 1$ $y - y_1 = m(x - x_1)$ $y - y_1 = 1(x - x_1)$ $y - 2 = 1(x - (-1))$ $y - 2 = x + 1$ $\therefore y = x + 3$ $y - x = 3$	<p>✓ m_{radius} ✓ m_{tangent}</p> <p>✓ substitute $(-1 ; 2)$ and m ✓ simplification (4)</p> <p>✓ m_{radius} ✓ m_{tangent}</p> <p>✓ substitute $(-1 ; 2)$ and m ✓ simplification (4)</p>
<p>4.4</p>	<p>Tangents to circle: $y = x + 3$ and $y = x + 1$</p> <p>$\therefore t > 3$ or $t < 1$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Answers only: Full marks</div>	<p>✓ $y = x + 1$</p> <p>✓ $t > 3$ ✓ $t < 1$ (3)</p>
<p>4.5</p>	<p>Draw rectangle CNED:</p> <p>Midpt of DN $(-\frac{7}{4}; \frac{3}{4})$</p> <p>$\therefore E(-\frac{5}{2}; -\frac{1}{2})$</p> <div style="text-align: center;">  </div> <p>OR/OF</p> <p>D(-3; 0)</p> <p>C → N: $(x; y) \rightarrow (x + 0,5; y - 0,5)$</p> <p>D → E: $D(x; y) \rightarrow E(x + 0,5; y - 0,5)$</p> <p>$\therefore E(-3 + 0,5; 0 - 0,5)$</p> <p>$\therefore E(-2,5; -0,5)$</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;">Answer only: Full marks</div>	<p>✓ midpt of DN</p> <p>✓ x value ✓ y value (3)</p> <p>✓ coordinates of D</p> <p>✓ x value ✓ y value (3)</p>

4.6



$$\begin{aligned} \text{area of trapezium AOBC} &= \frac{1}{2}(1+2)(1) \\ &= 1\frac{1}{2} \text{ square units} \end{aligned}$$

$$\begin{aligned} \text{area of } \triangle ACD &= \frac{1}{2}(2)(2) \\ &= 2 \text{ square units} \end{aligned}$$

$$\text{area of quadrilateral OBCD} = 3\frac{1}{2} \text{ square units}$$

$$\begin{aligned} \therefore 2a^2 &= \frac{7}{2} \\ a^2 &= \frac{7}{4} \\ a &= \frac{\sqrt{7}}{2} \end{aligned}$$

OR

✓ substitution into area of trapezium form

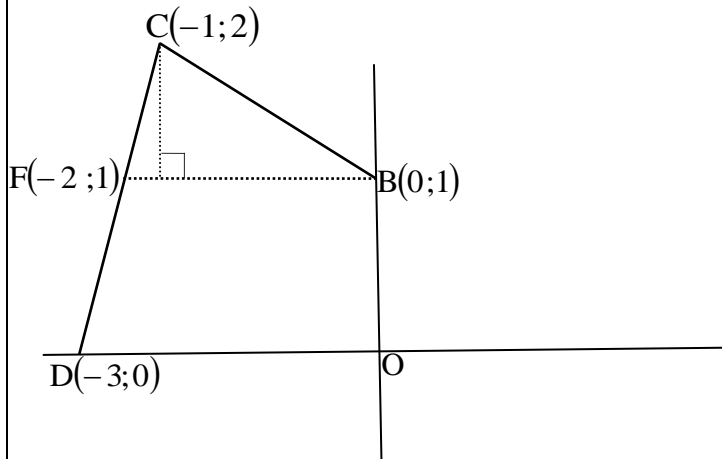
✓ area of trapezium

✓ area of triangle

✓ area of OBCD

✓ equating area OBCD to $2a^2$

(5)



BM produced cuts the tangent at F.

$$\begin{aligned} \text{area of } \triangle CFB &= \frac{1}{2}(2)(1) \\ &= 1 \text{ square unit} \end{aligned}$$

$$\begin{aligned} \text{area of trapezium BFDO} &= \frac{1}{2}(2+3)(1) \\ &= 2\frac{1}{2} \text{ square units} \end{aligned}$$

$$\text{area of quadrilateral OBCD} = 3\frac{1}{2} \text{ square units}$$

$$\therefore 2a^2 = \frac{7}{2}$$

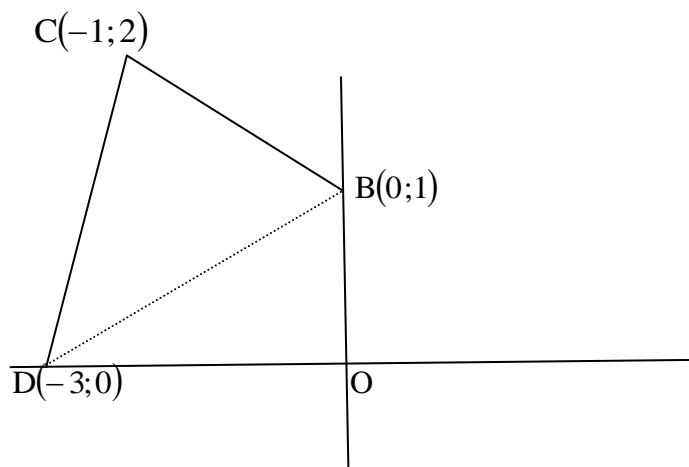
$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

OR

- ✓ area of triangle
- ✓ substitution into area of trapezium
- ✓ area of trapezium
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

(5)



Join DB

$$\begin{aligned} \text{area of } \triangle ODB &= \frac{1}{2}(3)(1) \\ &= \frac{3}{2} \text{ square unit} \end{aligned}$$

$$\begin{aligned} \text{area of } \triangle DCB &= \frac{1}{2}(2\sqrt{2})(\sqrt{2}) \\ &= 2 \text{ square unit} \end{aligned}$$

$$\therefore \text{area of OBCD} = \frac{3}{2} + 2 = \text{square units}$$

$$2a^2 = \frac{7}{2}$$

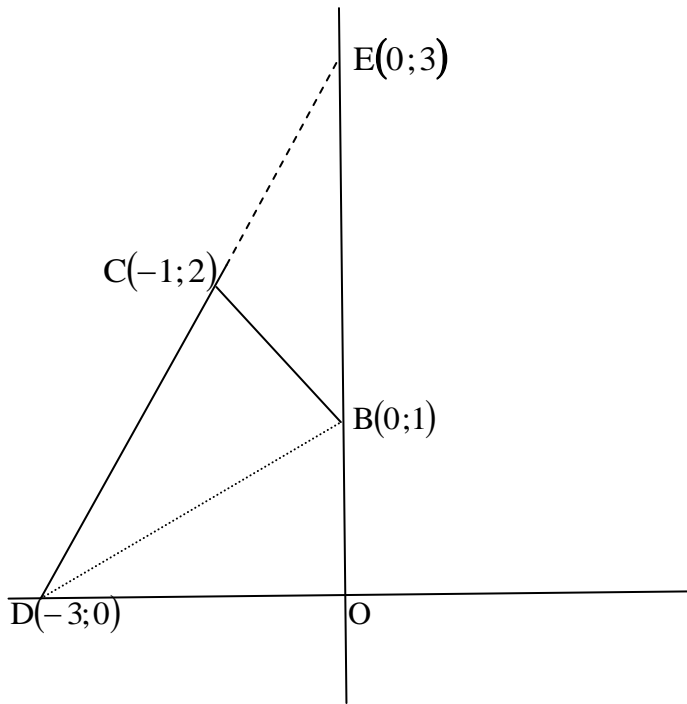
$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

OR

- ✓ area of Δ
- ✓ subst into area of Δ
- ✓ area of Δ
- ✓ area of OBCD
- ✓ equating area OBCD to $2a^2$

(5)



Let E be the point of intersection of DC with the positive y-axis.

$$\text{area of } \triangle DEO = \frac{1}{2} (3)(3)$$

$$= \frac{9}{2} \text{ square unit}$$

$$\text{area of } \triangle ECB = \frac{1}{2} (2)(1) \text{ or } \frac{1}{2} (\sqrt{2})(\sqrt{2})$$

$$= 1 \text{ square unit}$$

$$\text{area of quadrilateral OBCD} = \frac{9}{2} - 1 = 3\frac{1}{2} \text{ square units}$$

$$\therefore 2a^2 = \frac{7}{2}$$

$$a^2 = \frac{7}{4}$$

$$a = \frac{\sqrt{7}}{2}$$

✓ area of Δ

✓ subst into area of Δ

✓ area of Δ

✓ area of OBCD

✓ equating area
OBCD to $2a^2$

(5)

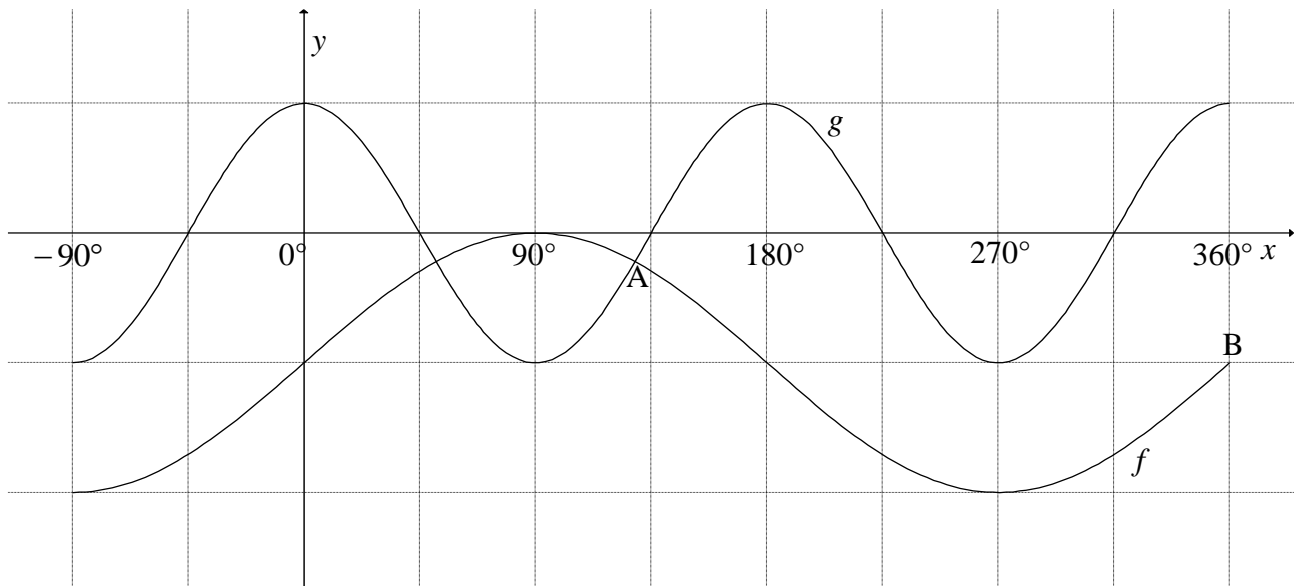
[20]

QUESTION/VRAAG 5

<p>5.1</p>	$\frac{\sin x}{\cos x \cdot \tan x} + \sin(180^\circ + x) \cos(90^\circ - x)$ $= \frac{\sin x}{\cos x \cdot \frac{\sin x}{\cos x}} + (-\sin x) \sin x$ $= 1 - \sin^2 x$ $= \cos^2 x$	<p>✓ $-\sin x$ ✓ $\sin x$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $1 - \sin^2 x$ ✓ $\cos^2 x$</p> <p>(5)</p>
<p>5.2</p>	$\frac{\sin^2 35^\circ - \cos^2 35^\circ}{4 \sin 10^\circ \cos 10^\circ}$ $= \frac{-(\cos^2 35^\circ - \sin^2 35^\circ)}{2(2 \sin 10^\circ \cos 10^\circ)}$ $= \frac{-\cos 70^\circ}{2 \sin 20^\circ}$ $= \frac{-\cos 70^\circ}{2 \cos 70^\circ} \quad \text{OR} \quad = \frac{-\sin 20^\circ}{2 \sin 20^\circ} = -\frac{1}{2}$	<p>✓ $-(\cos^2 35^\circ - \sin^2 35^\circ)$ ✓ $-\cos 70^\circ$ ✓ $2 \sin 20^\circ$</p> <p>✓ answer</p> <p>(4)</p>
<p>5.3</p>	$2 \sin^2 77^\circ = 2[\sin(90^\circ - 13^\circ)]^2$ $= 2 \cos^2 13^\circ$ $= 2 \cos^2 13^\circ - 1 + 1$ $= \cos 26^\circ + 1$ $= m + 1$ <p>OR</p> $1 - 2 \sin^2 77^\circ = \cos 154^\circ$ $2 \sin^2 77^\circ = 1 - \cos 154^\circ$ $= 1 - (-\cos 26^\circ)$ $= 1 + m$	<p>✓ using co-ratio ✓ reduction ✓ $2 \cos^2 13^\circ - 1 = \cos 26^\circ$ ✓ answer</p> <p>(4)</p> <p>✓ $1 - 2 \sin^2 77^\circ = \cos 154^\circ$ ✓ $2 \sin^2 77^\circ = 1 - \cos 154^\circ$ ✓ reduction ✓ answer</p> <p>(4)</p>
<p>5.4.1</p>	$\sin(x + 25^\circ) \cos 15^\circ - \cos(x + 25^\circ) \sin 15^\circ = \tan 165^\circ$ $\sin(x + 25^\circ - 15^\circ) = -0,2679... \text{ OR } -2 + \sqrt{3}$ $\sin(x + 10^\circ) = -0,2679... \text{ OR } -2 + \sqrt{3}$ $x + 10^\circ = 195,54^\circ + k \cdot 360^\circ \quad \text{or} \quad x + 10^\circ = 344,46^\circ + k \cdot 360^\circ$ $x = 185,54^\circ + k \cdot 360^\circ; k \in \mathbb{Z} \quad \text{or} \quad x = 334,46^\circ + k \cdot 360^\circ; k \in \mathbb{Z}$ <p>OR/OF</p>	<p>✓ ✓ $\sin(x + 10^\circ)$ ✓ $-0,2679...$ ✓ $195,54^\circ$ & $344,46^\circ$ ✓ $185,54^\circ$ & $334,46^\circ$ ✓ $+ k \cdot 360^\circ; k \in \mathbb{Z}$</p> <p>(6)</p>

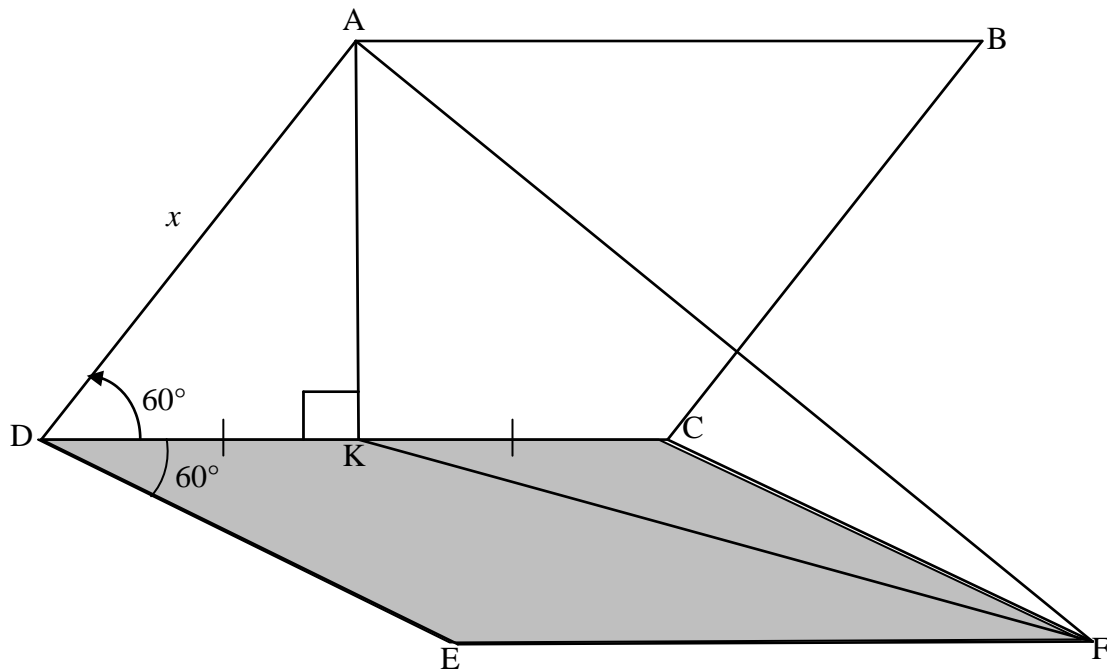
	$\sin(x + 25^\circ)\sin 75^\circ - \cos(x + 25^\circ)\cos 75^\circ = \tan 165^\circ$ $-(\cos(x + 25^\circ)\cos 75^\circ - \sin(x + 25^\circ)\sin 75^\circ) = -0,2679\dots$ $\cos(x + 100^\circ) = 0,2679\dots$ $\text{ref. } \angle = 74.4577\dots^\circ$ $x + 100^\circ = 74,46^\circ + k.360^\circ \text{ or } x + 100^\circ = 285,54^\circ + k.360^\circ$ $x = -25,54^\circ + k.360^\circ; k \in Z \text{ or } x = 185,54^\circ + k.360^\circ; k \in Z$	$\checkmark\checkmark \cos(x + 100^\circ)$ $\checkmark -0,2679\dots$ $\checkmark 74,46^\circ \text{ \& } 285,54^\circ$ $\checkmark -25,54^\circ \text{ \& } 185,54^\circ$ $\checkmark + k.360^\circ; k \in Z$ <p style="text-align: right;">(6)</p>
5.4.2	$f(x) = \sin(x + 10^\circ)$ <div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px 0;">Answers only: Full marks</div> <p>For minimum value of $\sin x$: $x = 270^\circ$ For minimum value of $\sin(x + 10^\circ)$: $x = 260^\circ$</p>	$\checkmark f(x) = \sin(x + 10^\circ)$ $\checkmark 270^\circ$ $\checkmark \text{ answer}$ <p style="text-align: right;">(3)</p>
		[22]

QUESTION/VRAAG 6



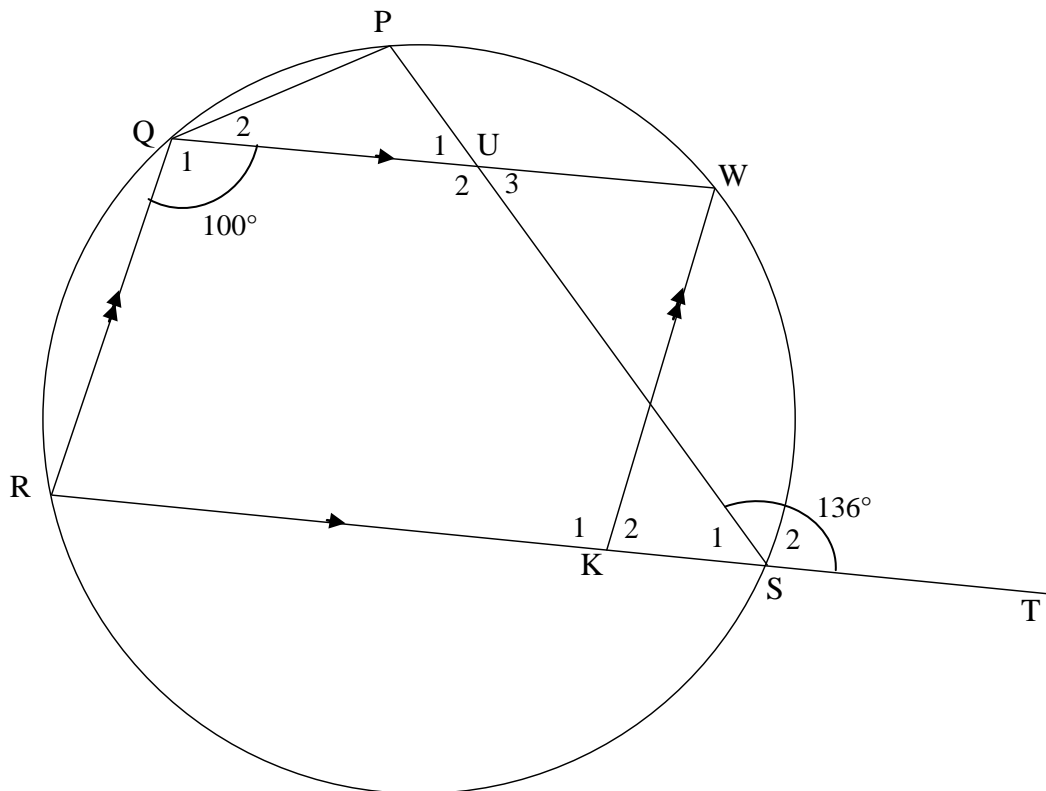
6.1	Range of f : $y \in [-2 ; 0]$ OR $-2 \leq y \leq 0$	✓ critical values ✓ notation (2)
6.2	$x \in (90^\circ ; 270^\circ)$ OR $x \in [90^\circ ; 270^\circ]$	✓ critical values ✓ notation (2)
6.3	$PQ = \cos 2x - (\sin x - 1)$ $= 1 - 2\sin^2 x - \sin x + 1$ $= -2\sin^2 x - \sin x + 2$ $\sin x = -\frac{b}{2a}$ $= \frac{-(-1)}{2(-2)}$ $\sin x = -\frac{1}{4}$ $\therefore x = 194,48^\circ \text{ or } x = 345,52^\circ$	✓ $PQ = \cos 2x - (\sin x - 1)$ ✓ $\cos 2x = 1 - 2\sin^2 x$ ✓ substitution into formula ✓ $\sin x = -\frac{1}{4}$ ✓ $194,48^\circ$ ✓ $345,52^\circ$ (6)
[10]		

QUESTION/VRAAG 7



7.1	$\sin 60^\circ = \frac{AK}{x}$ $AK = x \sin 60^\circ \text{ or } \frac{\sqrt{3}}{2}x \text{ or } 0,866x$	✓ trig ratio ✓ answer (2)
7.2	$\hat{KCF} = 120^\circ$	✓ answer (1)
7.3	$KF^2 = CF^2 + CK^2 - 2CF \cdot CK \cos \hat{KCF}$ $= x^2 + \left(\frac{x}{2}\right)^2 - 2x\left(\frac{x}{2}\right) \cos 120^\circ$ $= x^2 + \frac{x^2}{4} - x^2\left(-\frac{1}{2}\right)$ $= \frac{7x^2}{4}$ $KF = \frac{\sqrt{7}x}{2}$ $\hat{AKF} = y$ $\text{Area } \triangle AKF = \frac{1}{2} \cdot AK \cdot KF \sin \hat{AKF}$ $= \frac{1}{2} \cdot \frac{\sqrt{3}x}{2} \cdot \frac{\sqrt{7}x}{2} \sin y$ $= \frac{x^2 \sqrt{21} \sin y}{8}$	✓ correct use of cosine rule ✓ substitution ✓ $\cos 120^\circ = -\frac{1}{2}$ ✓ $KF = \frac{\sqrt{7}x}{2}$ ✓ correct use of area rule ✓ substitution ✓ answer in terms of x and y (7)
[10]		

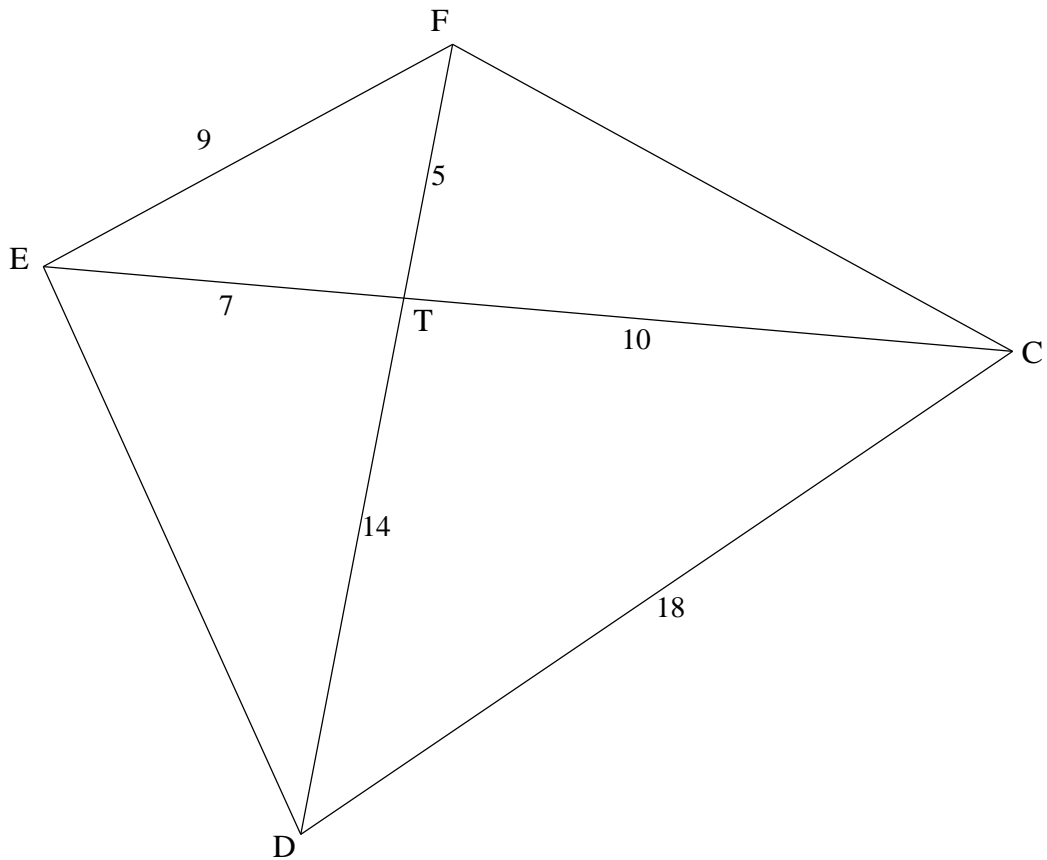
QUESTION/VRAAG 8



8.1.1	$\hat{R} = 80^\circ$ [co-int \angle s/ko-binne \angle e; QW RK]	\checkmark S \checkmark R (2)
8.1.2	$\hat{P} = 100^\circ$ [opp \angle s of cyclic quad/teenoorst \angle e v koordevh]	\checkmark S \checkmark R (2)
8.1.3	$\hat{PQR} = 136^\circ$ [ext \angle of cyclic quad/buite \angle v koordevh] $\hat{Q}_2 = 36^\circ$ OR $\hat{P}UW = \hat{S}_2 = 136^\circ$ [corresp \angle s/ooreenkomstige \angle e; QW RK] $\hat{P}QW + \hat{P} = \hat{P}UW$ [ext \angle s of/buite \angle van Δ QPU] $\hat{P}QW + 100^\circ = 136^\circ$ $\hat{P}QW = 36^\circ$ OR $\hat{U}_3 = 180^\circ - 136^\circ = 44^\circ$ [co-int \angle s/ko-binne \angle e; QW RK] $\hat{U}_1 = \hat{U}_3 = 44^\circ$ [vert opp \angle s/regoorstaande \angle e] $\hat{P}QW = 180^\circ - (100 + 44^\circ)$ [sum of \angle s in Δ /som \angle e van Δ] $\hat{P}QW = 36^\circ$	\checkmark S \checkmark R \checkmark S (3) \checkmark S \checkmark R \checkmark S (3)

8.1.4	$\hat{U}_2 = \hat{S}_2 = 136^\circ$ [alt \angle s/verwiss \angle e ; QW RK] OR $\hat{U}_2 = 100^\circ + 36^\circ$ [ext \angle s of/buite \angle van Δ QPU] $= 136^\circ$ OR $\hat{U}_2 = P\hat{U}W = 136^\circ$ [vert opp \angle s/regoorstaande \angle e] OR $\hat{U}_2 = 180^\circ - \hat{U}_3$ [\angle s on a str line/ \angle e op reguitlyn] $= 180^\circ - 44^\circ$ $= 136^\circ$	\checkmark S \checkmark R (2) \checkmark S \checkmark R (2) \checkmark S \checkmark R (2) \checkmark S \checkmark R (2)
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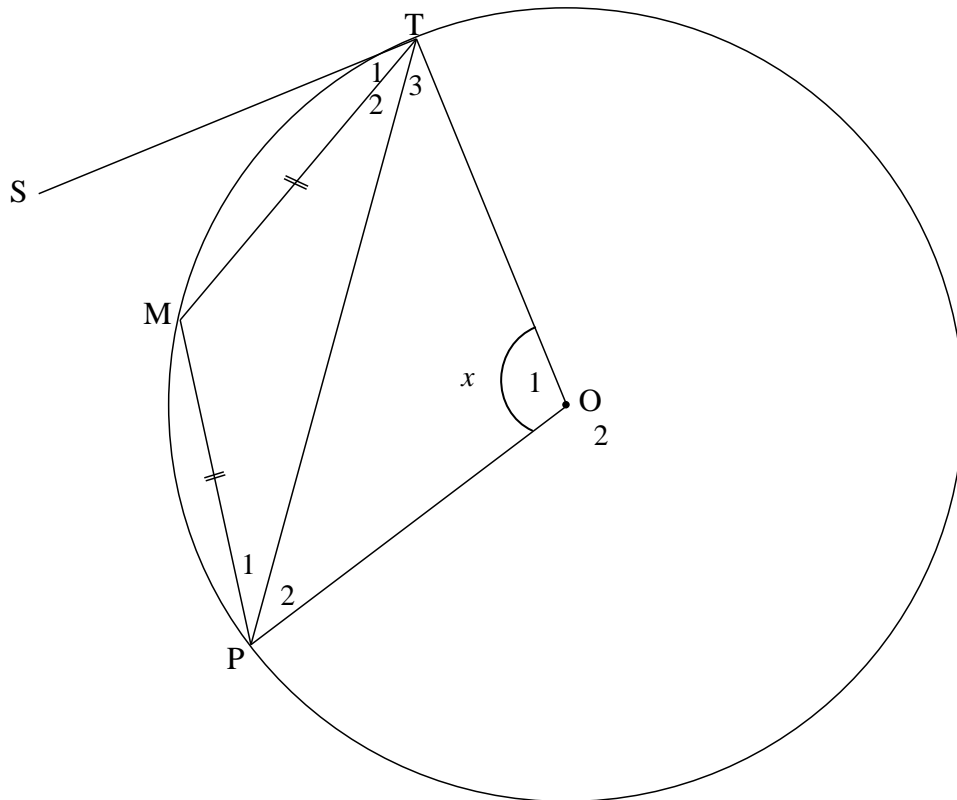
8.2



<p>8.2.1</p>	<p>In $\triangle EFT$ and $\triangle DCT$:</p> $\frac{EF}{CD} = \frac{9}{18} = \frac{1}{2}$ $\frac{FT}{TC} = \frac{5}{10} = \frac{1}{2}$ $\frac{ET}{TD} = \frac{7}{14} = \frac{1}{2}$ <p>$\therefore \triangle EFT \parallel \triangle DCT$ [Sides of Δ in prop/ sye van Δ in dieselfde verh]</p> <p>$\therefore \hat{EFD} = \hat{ECD}$</p> <p>OR</p> <p>In $\triangle FET$: $49 = 25 + 81 - 2(5)(9)\cos\hat{F}$ $\cos\hat{F} = \frac{19}{30}$ $\hat{F} = 50,7^\circ$</p> <p>In $\triangle TDC$: $196 = 100 + 256 - 2(10)(18)\cos\hat{C}$ $\cos\hat{C} = \frac{19}{30}$ $\hat{C} = 50,7^\circ$</p>	<p>$\checkmark\checkmark$ all 3 ratios = $\frac{1}{2}$</p> <p>$\checkmark \triangle EFT \parallel \triangle DCT \checkmark R$</p> <p>(4)</p> <p>$\checkmark\checkmark \hat{F} = 50,7^\circ$</p> <p>$\checkmark\checkmark \hat{C} = 50,7^\circ$</p> <p>(4)</p>
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<p>8.2.2</p>	<p>$\hat{E}FD = \hat{E}CD$ [proved in 8.2.1] E, F, C and D are concyclic EFCD is a cyclic quad [converse \angles in the same segment/ <i>omgekeerde \anglee in dies segment</i>] $\therefore \hat{D}FC = \hat{D}EC$ [\angles in the same segment/\anglee in dies segment]</p>	<p>\checkmarkS \checkmarkR \checkmark R</p> <p style="text-align: right;">(3)</p>
[16]		

QUESTION/VRAAG 9

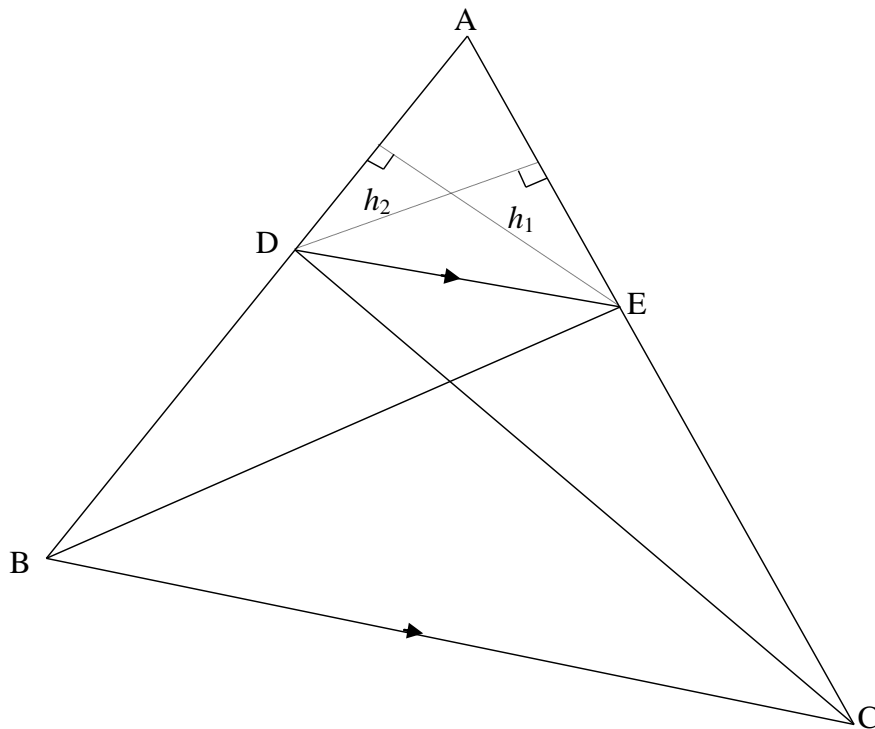


$\hat{O}_2 = 360^\circ - x \quad [\angle\text{s round a pt}/\angle\text{e om 'n punt}]$ $\therefore \hat{M} = 180^\circ - \frac{1}{2}x \quad [\angle \text{at centre} = 2 \times \angle \text{at circumference}/$ <p style="text-align: center;"><i>middelpunts</i> $\angle = 2 \times \text{omtreks}$ \angle]</p> $\therefore \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x \quad [\text{sum of } \angle\text{s in } \Delta/\text{som } \angle\text{e van } \Delta]$ $\therefore \hat{T}_2 = \hat{P}_1 = \frac{1}{4}x \quad [\angle\text{s opp equal sides}/\angle\text{e teenoor gelyke sye}]$ $\therefore \hat{STM} = \hat{P}_1 = \frac{1}{4}x \quad [\text{tan chord theorem}/\text{raaklyn koordstelling}]$	$\checkmark \hat{O}_2 = 360^\circ - x$ $\checkmark \hat{M} = 180^\circ - \frac{1}{2}x \quad \checkmark R$ $\checkmark \hat{T}_2 + \hat{P}_1 = \frac{1}{2}x$ $\checkmark \hat{P}_1 = \frac{1}{4}x \quad \checkmark R$ $\checkmark R$ <p style="text-align: right;">(7)</p>
<p>OR/OF</p> $\hat{O}_2 = 360^\circ - x \quad [\angle\text{s round a pt}/\angle\text{e om 'n punt}]$ $\therefore \hat{M} = \frac{1}{2}\hat{O}_2 \quad [\angle \text{at centre} = 2 \times \angle \text{at circumference}]$ $\therefore \hat{T}_2 + \hat{P}_1 = 180^\circ - \hat{M} \quad [\text{sum of } \angle\text{s in } \Delta/\text{som } \angle\text{e van } \Delta]$ $\therefore \hat{T}_2 = \hat{P}_1 \quad [\angle\text{s opp equal sides}/\angle\text{e teenoor gelyke sye}]$ $= \frac{180^\circ - \hat{M}}{2} = \frac{180^\circ - \frac{1}{2}\hat{O}_2}{2} = \frac{180^\circ - \frac{1}{2}(360^\circ - x)}{2} = \frac{1}{4}x$ $\therefore \hat{STM} = \frac{1}{4}x \quad [\text{tan chord theorem}/\text{raaklyn koordstelling}]$	$\checkmark \hat{O}_2 = 360^\circ - x$ $\checkmark S \quad \checkmark R$ $\checkmark S$ $\checkmark R$ $\checkmark S$ $\checkmark R$ <p style="text-align: right;">(7)</p>

[7]

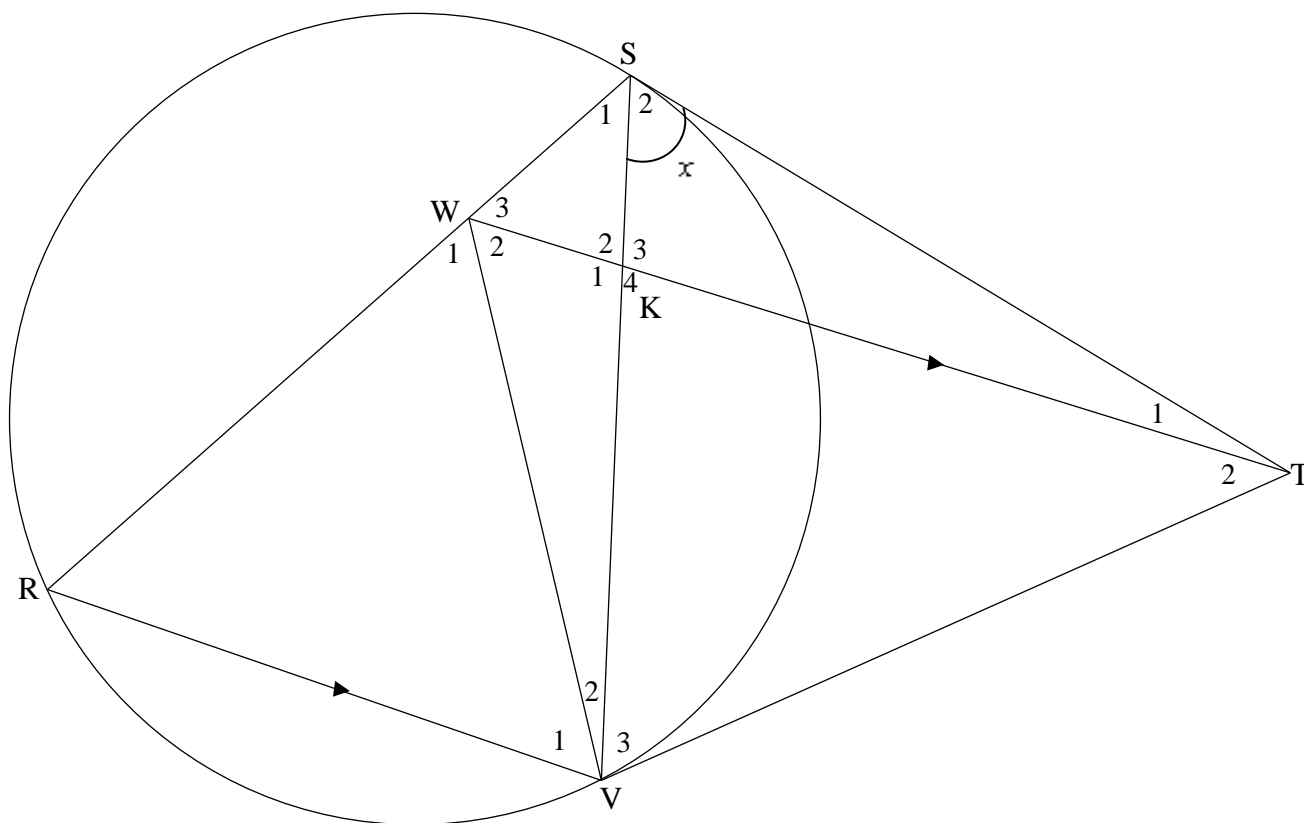
QUESTION/VRAAG 10

10.1



<p>10.1</p>	<p>Constr: Draw h_1 from E \perp AD and h_2 from D \perp AE <i>Konstr: Trek h_1 vanaf E \perp AD en h_2 vanaf D \perp AE</i></p> <p>Proof/Bewys:</p> $\frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1} = \frac{AD}{DB}$ $\frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{\frac{1}{2}AE \times h_2}{\frac{1}{2}EC \times h_2} = \frac{AE}{EC}$ <p>But area $\triangle BDE = \text{area } \triangle DEC$ [same base & height or DE \parallel BC/ <i>dies basis & hoogte; of DE \parallel BC]</i></p> $\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	<p>✓ constr/konstr OR reason: common vertex or same height</p> $\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2}AD \times h_1}{\frac{1}{2}DB \times h_1}$ $\checkmark \frac{\text{area } \triangle ADE}{\text{area } \triangle DEC} = \frac{AE}{EC}$ <p>✓ S ✓ R ✓ S</p> <p style="text-align: right;">(6)</p>
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10.2



10.2.1	$\hat{V}_3 = x$ [Tans from same point/raaklyne vanaf dieselfde pt] $\hat{R} = x$ [tan chord theorem/raaklyn koordstelling] $\hat{W}_3 = x$ [corresp \angle s/ooreenkomstige \angle e; WT RV]	✓ S ✓ R ✓ S ✓ R ✓ S ✓ R (6)
10.2.2(a)	$\hat{V}_3 = \hat{W}_3 = x$ [proved in 10.2.1] W, S, T and V are concyclic/is konsiklies WSTV is a cyclic quad [converse \angle s in the same segment/ Omgekeerde \angle e in dieselfde segment]	✓ S ✓ R (2)
10.2.2(b)	$\hat{W}_2 = \hat{S}_2 = x$ [\angle s in the same segment/ \angle e in dies segment] $\hat{V}_1 = \hat{W}_2 = x$ [alt \angle s/verwiss \angle e ; WT RV] But $\hat{R} = x$ [proved in 10.2.1] $\therefore \hat{R} = \hat{V}_1 = x$ $\therefore WR = WV$ [sides opp equal \angle s/sye teenoor gelyke \angle e] ΔWRV is isosceles/is gelykbenig OR/OF	✓ S ✓ R ✓ S/ R ✓ S (4)

	$\hat{S}_2 = \hat{W}_2 = x$ [\angle s in the same segment] $\hat{W}_2 = \hat{W}_3 = x$ $\hat{W}_2 + \hat{W}_3 = \hat{R} + \hat{V}_1$ [ext \angle of Δ] $\therefore \hat{V}_1 = x = \hat{R}$ $\therefore WR = WV$ [sides opp equal \angle s/sye teenoor gelyke \angle e] ΔWRV is isosceles/is gelykbenig	✓ S ✓R ✓ S/R ✓ S (4)
10.2.2(c)	In ΔWRV and/en ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved OR tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\therefore \Delta WRV \parallel \Delta TSV$ [\angle, \angle, \angle] OR/OF In ΔWRV and/en ΔTSV $\hat{R} = \hat{S}_2 = x$ [proved OR tan chord theorem] $\hat{V}_1 = \hat{V}_3 = x$ [proved] $\hat{W}_1 = \hat{S}_1 = x$ [sum of \angle s in Δ / \angle e van Δ] $\therefore \Delta WRV \parallel \Delta TSV$	✓ S ✓ S ✓ R (3) ✓ S ✓ S ✓ S (3)
10.2.2(d)	$\frac{RV}{SV} = \frac{WR}{TS}$ [$\Delta WRV \parallel \Delta TSV$] $\therefore WR \times SV = RV \times TS$ $\frac{WR}{SR} = \frac{KV}{SV}$ [prop theorem/eweredighst; $WT \parallel RV$] $\therefore WR \times SV = KV \times SR$ $\therefore RV \times TS = KV \times SR$ $\therefore \frac{RV}{SR} = \frac{KV}{TS}$ OR/OF In ΔRVS and/en ΔVKT $\hat{S}_1 = \hat{K}_4$ [alt \angle s, $WT \parallel RV$] $\hat{S}_1 = \hat{V}_3$ [proven] $\Delta RVS \parallel \Delta VKT$ [\angle, \angle, \angle] $\therefore \frac{RV}{SR} = \frac{KV}{VT}$ but $VT = ST$ [tans from same point] $\therefore \frac{RV}{SR} = \frac{KV}{TS}$	✓ correct ratios ✓ $\frac{WR}{SR} = \frac{KV}{SV}$ ✓ R ✓ equating $WR \times SV$ (4) ✓ identifying correct Δ s ✓ proving \parallel ✓ correct ratio ✓ S (4)
[25]		

TOTAL/TOTAAL: 150